Theory of machine

If you have a smart project, you can say "I'm an engineer"

Lecture 3

Instructor

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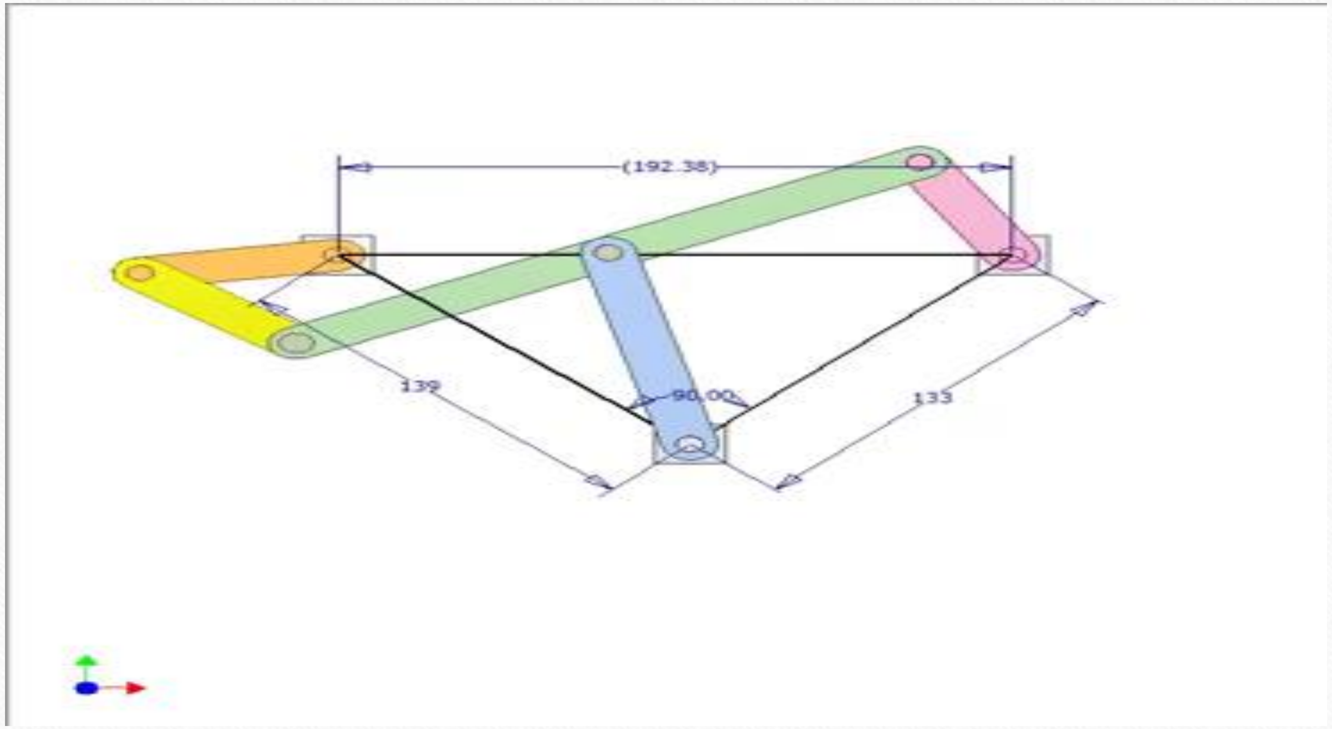
Theory of machine
MDP 234

- **Lecture aims:**
  - Understand the Kinematics analysis.
  - Identify the velocity diagram.

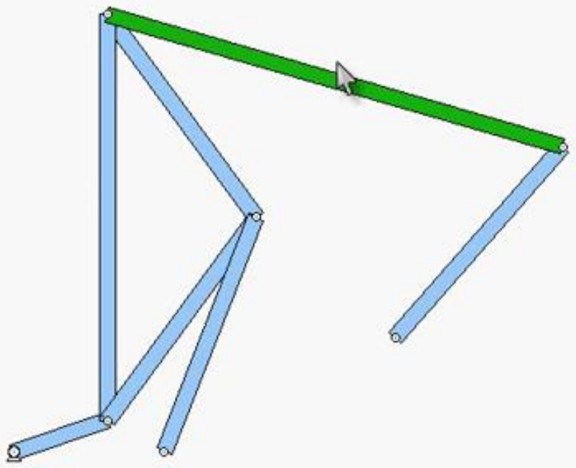
Model Examples

- Six bar mechanism



Model Examples

- Six bar mechanism



Planar Rigid body Kinematics: Review

- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.
- Classification of rigid body motions:

  - **translation**:
    - curvilinear translation
    - rectilinear translation

  - motion about a fixed point
  - general plane motion
  - rotation about a fixed axis
  - general motion

Planar Rigid body Kinematics: Review

- **Translation**
  - For any two particles in the body,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

  - Differentiating with respect to time,

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

$$\vec{v}_B = \vec{v}_A$$

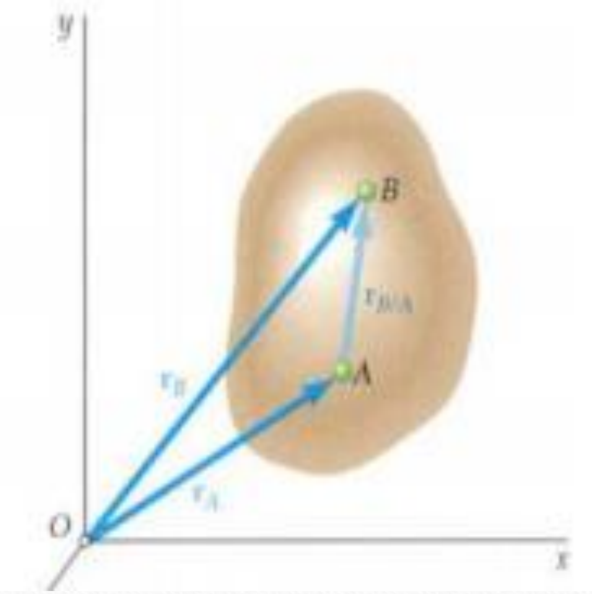
All particles have the same velocity.

  - Differentiating with respect to time again,

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

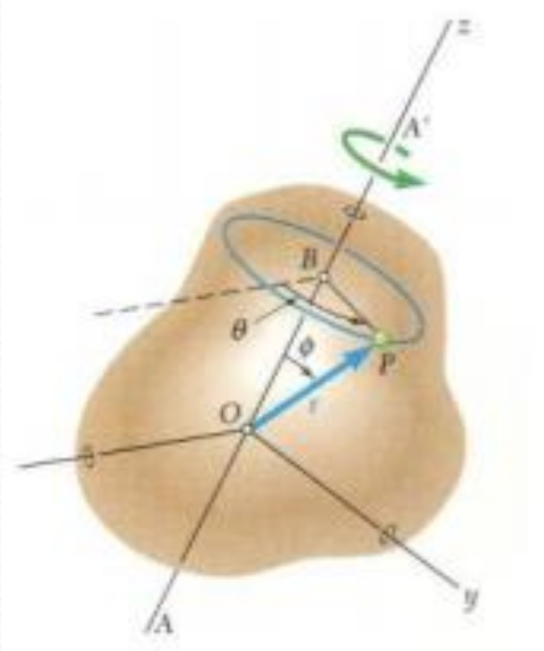
$$\vec{a}_B = \vec{a}_A$$

All particles have the same acceleration.



Planar Rigid body Kinematics: Review

- **Rotation About a Fixed Axis. Velocity**
  - Consider rotation of rigid body about a fixed axis *AA'*



Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle P is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

  - The same result is obtained from

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \text{angular velocity}$$

Planar Rigid body Kinematics: Review

- **Rotation About a Fixed Axis. Representative Slab**
- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.
- Velocity of any point *P* of the slab,

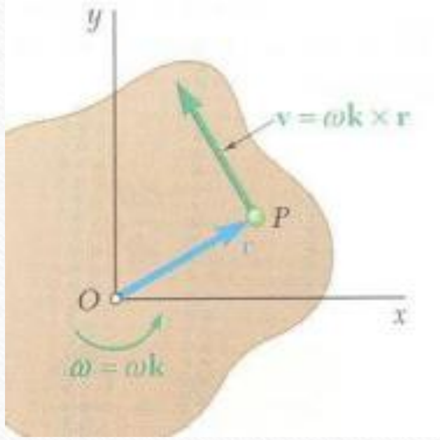
$$\vec{v} = \vec{\omega} \times \vec{r} = \omega \vec{k} \times \vec{r}$$
$$v = r\omega$$

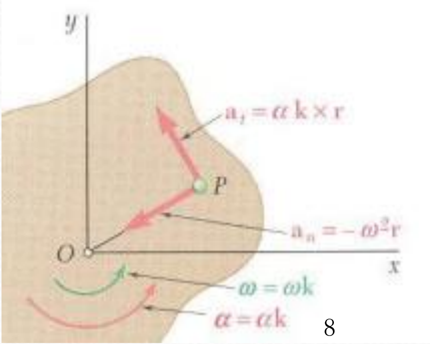
- Acceleration of any point *P* of the slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$
$$= \alpha \vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha \vec{k} \times \vec{r} \qquad a_t = r\alpha$$
$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$

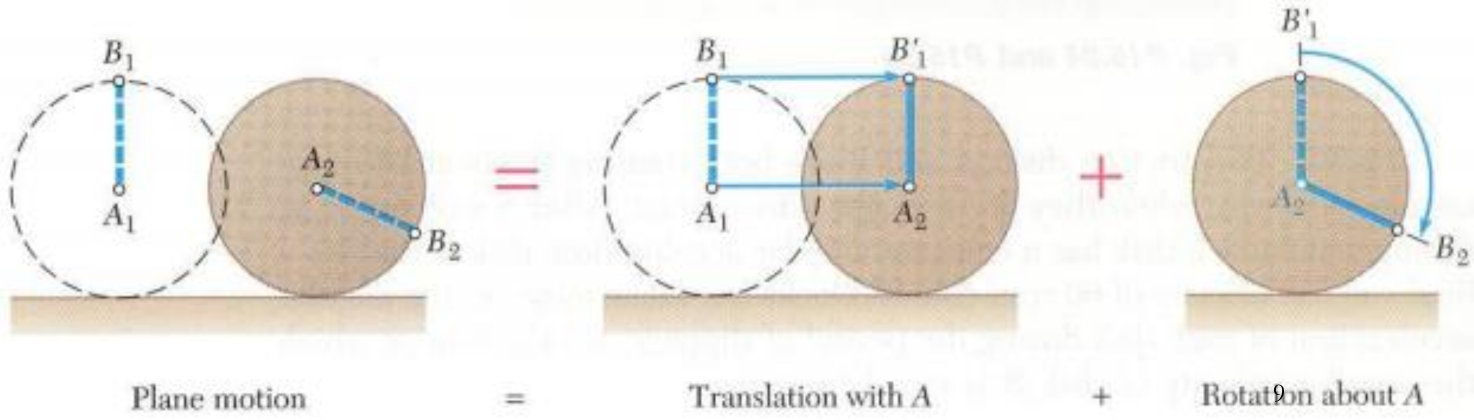




General Plane Motion

- *General plane motion* is neither a translation nor a rotation.
- General plane motion can be considered as the *sum* of a **translation and rotation**.
- Displacement of particles *A* and *B* to *A*2 and *B*2 can be divided into two parts:

- translation to *A*2 and *B*1'
- rotation of *B*1' about *A*2 to *B*2



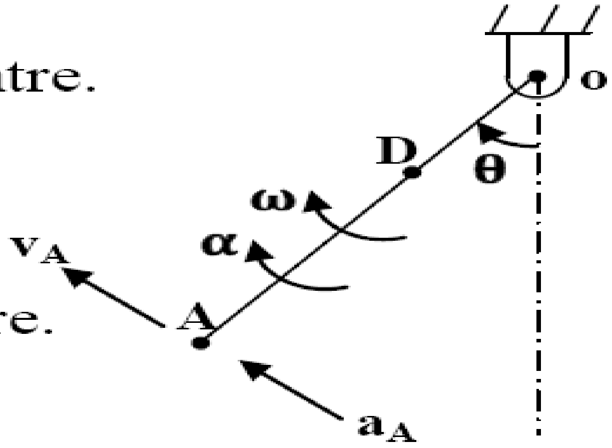
Bodies rotate about fixed point

Consider the link shown which is rotate about the fixed point o, the motion of this link can be analyzed using the principle of absolute motion as follow:

If θ: angular displacement about fixed rotation centre.

ω: angular velocity about fixed rotation centre.

α: angular acceleration about fixed rotation centre.



Bodies under general plane motion

The motion of any point can be discretized into translation and rotation, if consider the link shown under general plane motion, the ends , B of absolute velocities v_A, v_B, and absolute accelerations a_A, a_B then:-

$$\vec{V}_A = \vec{V}_B + \vec{V}_{AB} \qquad or \qquad \vec{V}_B = \vec{V}_A + \vec{V}_{BA}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{AB} \qquad or \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

Bodies under general plane motion

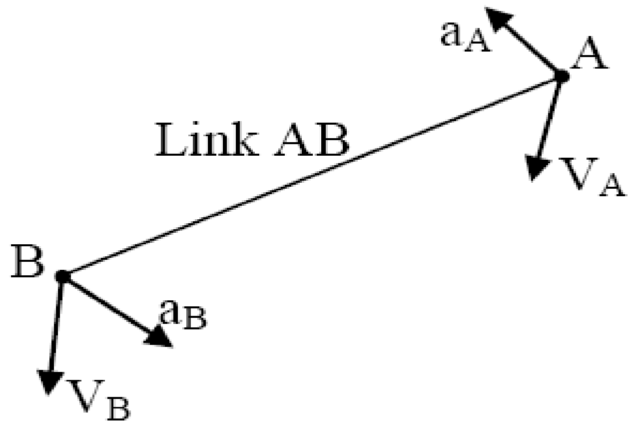
Where:-

$\vec{V}_{BA}$ is the relative velocity of B w.r.t A.

$\vec{V}_{AB}$ is the relative velocity of A w.r.t B.

$\vec{a}_{BA}$ is the relative acceleration of B w.r.t A.

$\vec{a}_{AB}$ is the relative acceleration of A w.r.t B.



Bodies under general plane motion

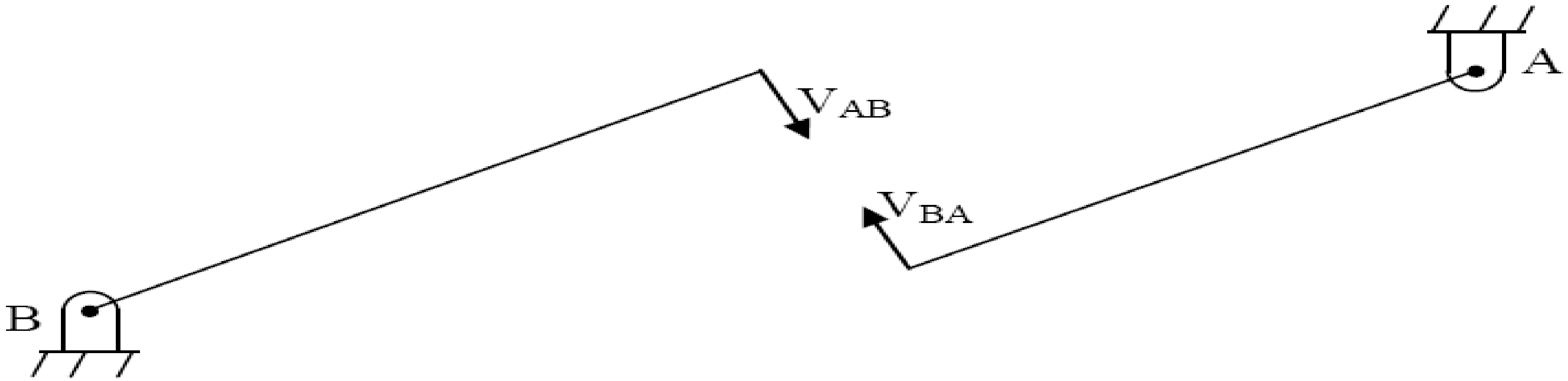
i.e the state of velocity can be replaced by one of the following:-

$$\left.\begin{aligned}\vec{V}_A &= \vec{V}_B + (\vec{w}) \times (\overrightarrow{AB}) \\ \vec{V}_B &= \vec{V}_A + (\vec{w}) \times (\overrightarrow{BA})\end{aligned}\right\} \quad \text{vector notation.}$$

Bodies under general plane motion

V_{AB} : mean that B is a fixed rotation a center, and A moved a round B

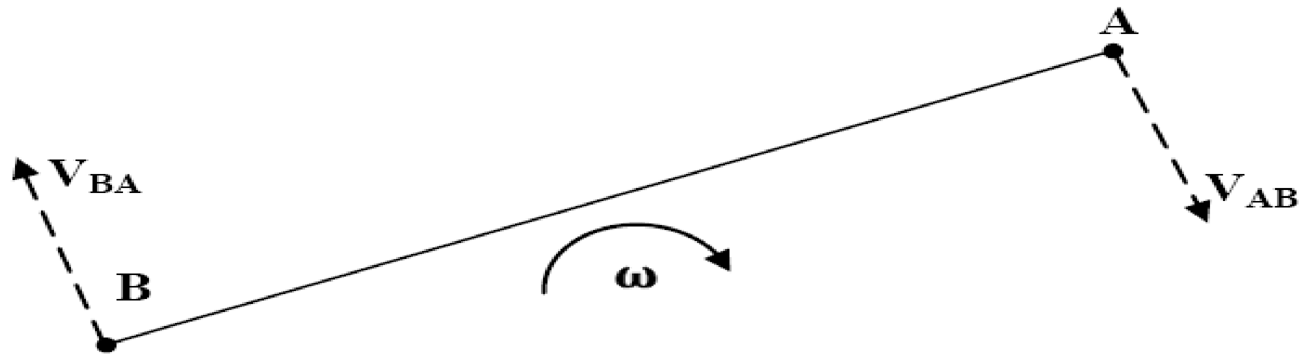
V_{BA} : mean that A is a fixed rotation a center, and B moved a round A

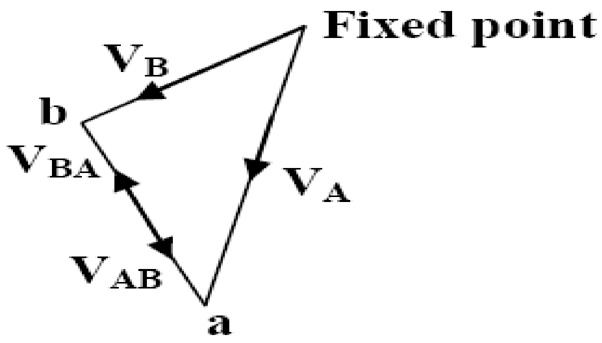


Bodies under general plane motion

$$\because \quad \omega_{AB} = \frac{V_{AB}}{AB} = \frac{V_{BA}}{AB}$$

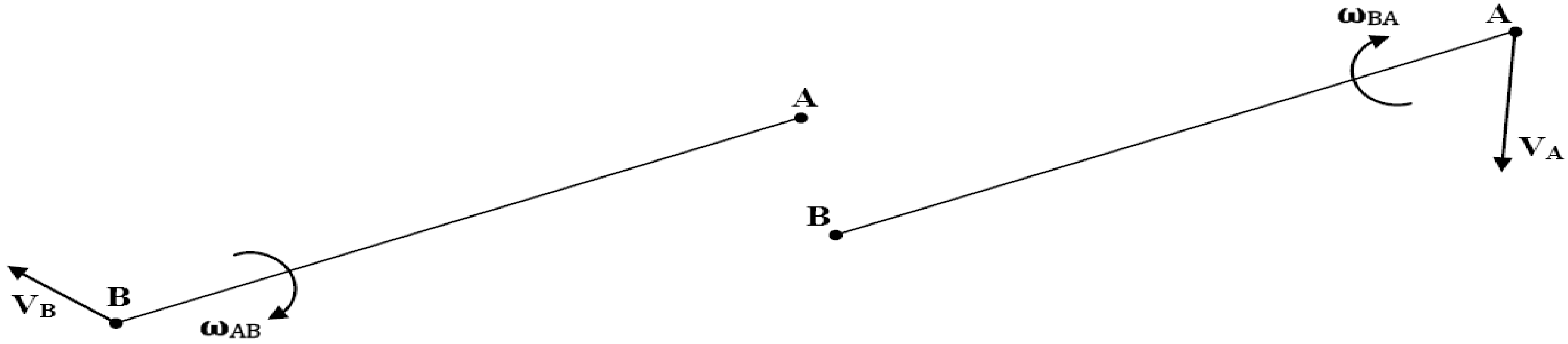
To specify direction of ω: $\Longrightarrow$





Bodies under general plane motion

∵ V_{AB} and V_{BA} ⊥ AB.



Velocity diagram

- **Consider the shown link under general plane motion, to specify the velocity of any point, it's required one of following:-**

- 1- * Absolute velocity of any point (value **and direction).**

  *Absolute velocity of other point (value **or direction).**
- 2- *Absolute velocity of any point (value **and direction).**

  *Angular velocity of the link which is the same for all points.

Velocity diagram

- Consider two points A and B on a rigid link AB, as shown in Fig 3 (a).
  - Let one of the extremities (B) of the link move relative to A , in a clockwise direction.
  - Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB.
  - It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.

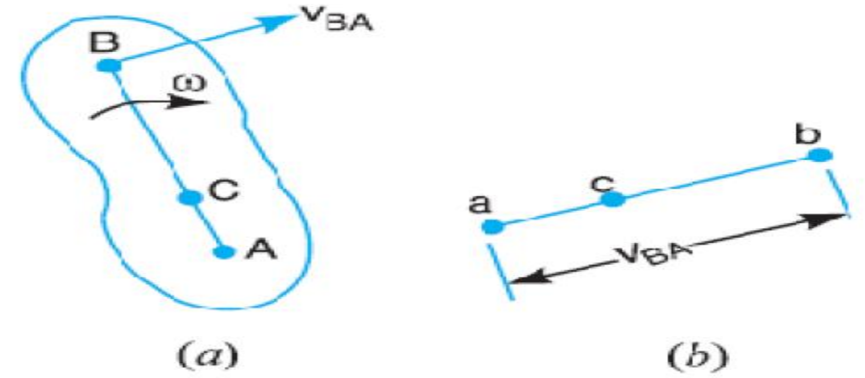


Fig. 3. Motion of a Link.

Velocity diagram

Hence velocity of any point on a link with respect to another point on the same link is always **perpendicular** to the **line joining** these points on the configuration (or space) diagram

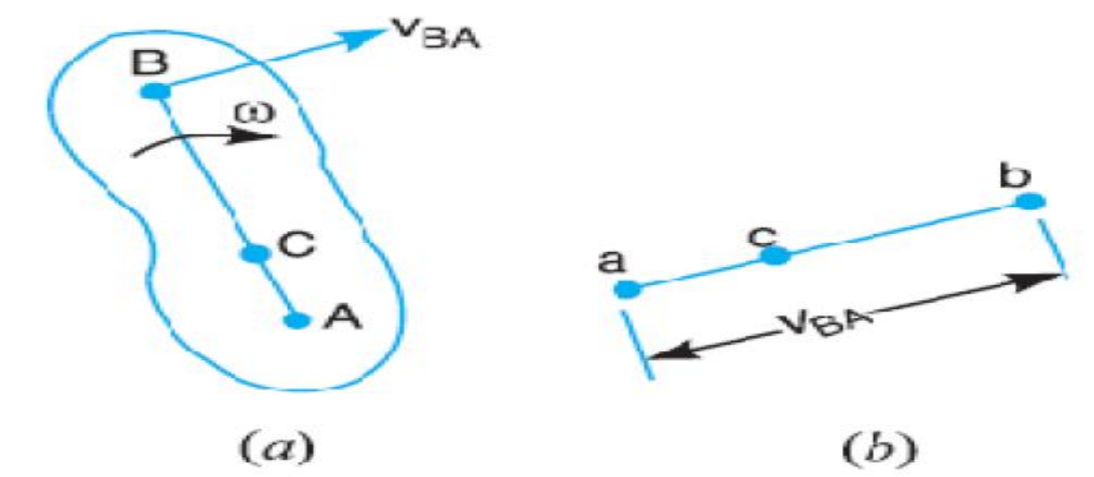


Fig. 3. Motion of a Link.

Velocity diagram

The relative velocity of B with respect to A (*i.e.* v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig. 3 (*b*).

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \quad \text{...(i)}$$

Similarly, the velocity of any point C on AB with respect to A,

$$v_{CA} = \overline{ac} = \omega . AC \quad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \quad \text{...(iii)}$$

Velocity diagram

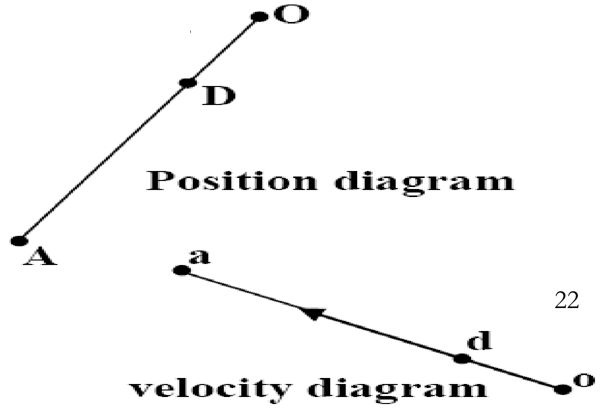
- Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.
- Note: The relative velocity of A with respect to B is represented by ba, although A may be a fixed point.
- The motion between A and B is only relative. Moreover, it is immaterial whether the link moves about A in a clockwise direction or about B in a clockwise direction.
- Velocity of a Point on a Link by Relative Velocity Method The relative velocity method is based upon the relative velocity of the various points of the link.

Velocity diagram

In order to analyze the velocity of any point we follow with one of following methods:

1- If ω is given:-

- Draw the link by SFp (scale factor for position), SFp=$\frac{drawn\ length\ of\ link}{actual\ length\ of\ link}$
- v_A=$(\omega)(oA)$.



Velocity diagram

- Select SFv $= \frac{oa}{v_A}$, then select a reference point of zero velocity, such as o.
- Draw from o, a line of length $(oa) = v_A.SFv \perp oA$ in direction of ω.
- To find the velocity of any point located on the link, such as D, specify point d on oa such that $\frac{od}{oa} = \frac{oD}{oA}$ $\Longrightarrow$ od=$\left(\frac{oD}{oA}\right) oa$.

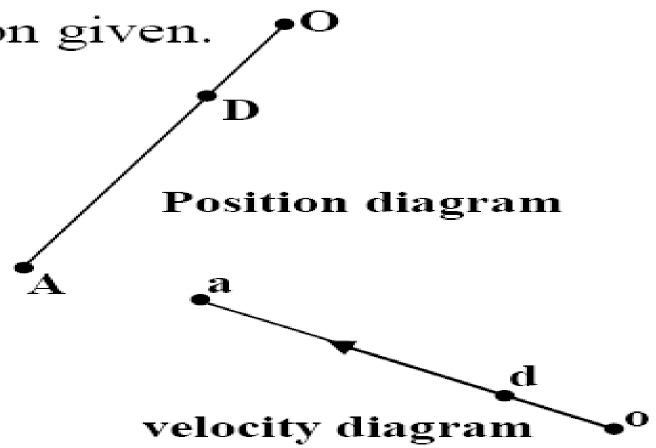
Then:- $v_D = \frac{od}{SFv}$.

Velocity diagram

2- If v_A is given:-

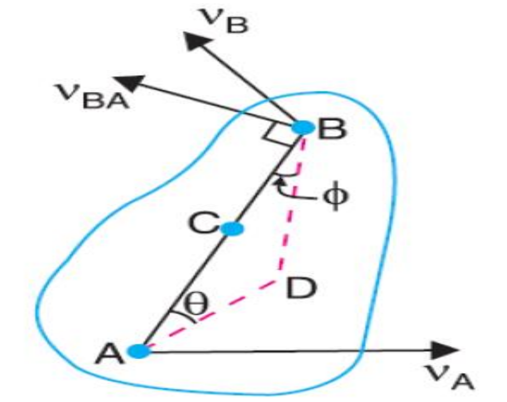
- Select SFv, specify reference point of zero velocity.
- Draw oa of length $(v_A)(SFv)$ in the same direction given.
- To find value and direction of ω:

Value of $\omega = \frac{v_A}{oA}$ ↻ .



Velocity diagram

- Consider two points A and B on a link as shown in Fig..4 (a).
- Let the absolute velocity of the point A i.e. vA is known in magnitude and direction
- the absolute velocity of the point B i.e. vB is known in direction only.
- Then the velocity of B may be determined by drawing the velocity diagram as shown in Fig. 4 (b).



(*a*) Motion of points on a link.

Velocity diagram

- **The velocity diagram is drawn as follows :**

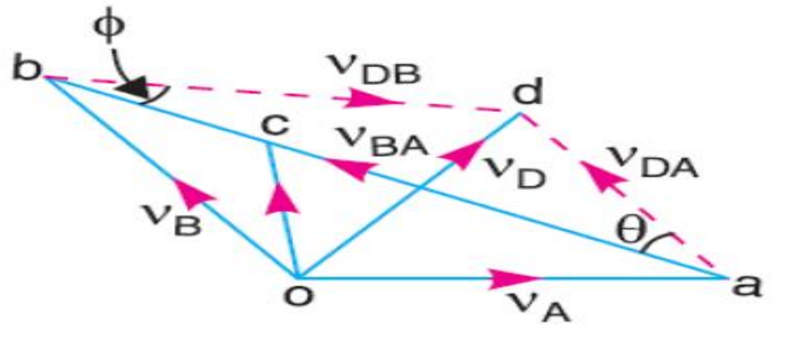
1. Take some convenient point o, known as the pole.

2. Through o, draw oa parallel and equal to vA, to some suitable scale.

3. Through a, draw a line perpendicular to AB of Fig. 4 (a). This line will represent the Velocity of B with respect to A , i.e. vBA

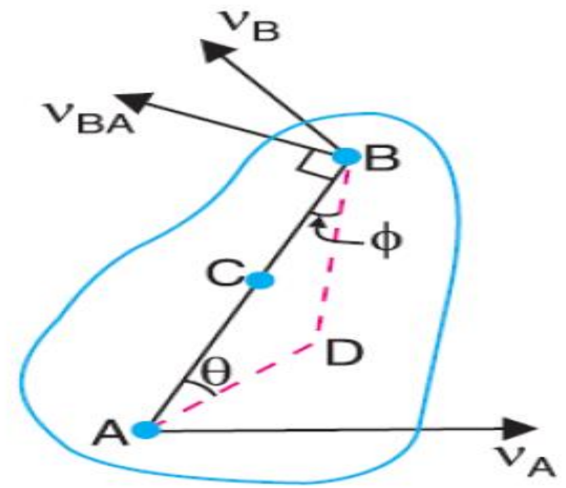
4. Through o, draw a line parallel to vB intersecting the line of vBA at b.

5. Measure ob, which gives the required velocity of point B (vB), to the scale Velocities



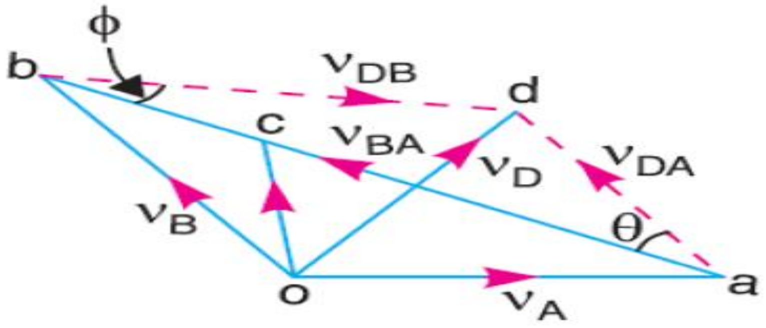
(*b*) Velocity diagram.

Velocity diagram



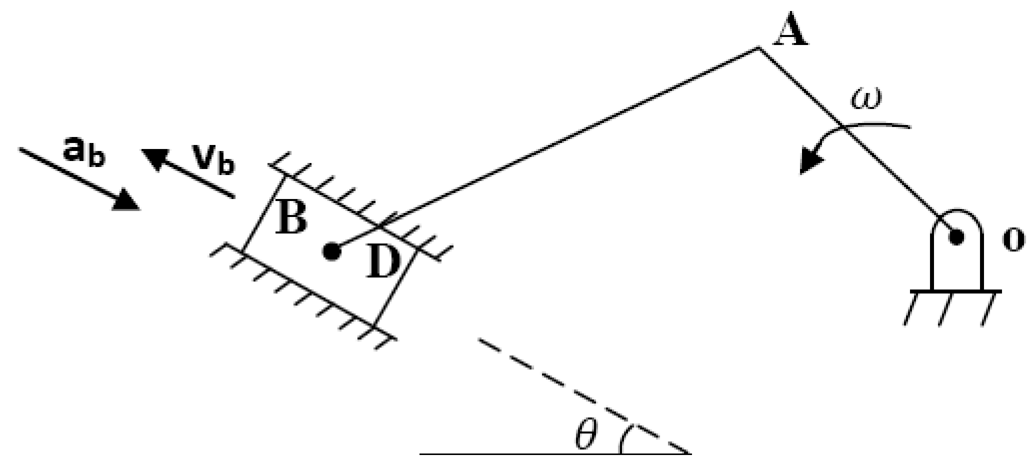
(*a*) Motion of points on a link.

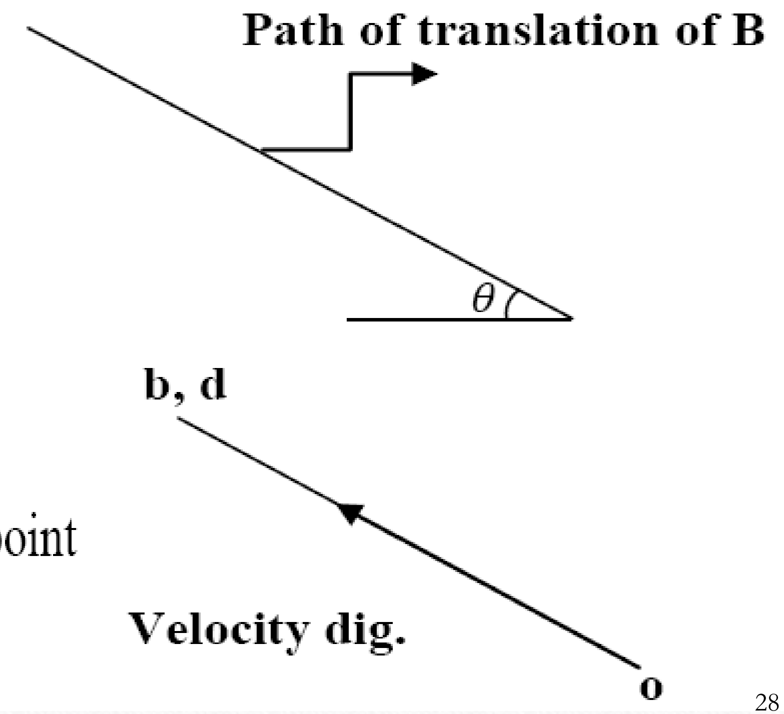
Fig. 4



(*b*) Velocity diagram.

Translated bodies Velocity diagram





Then all points on the piston have the same velocity, such as point D, i.e on the velocity diagram, the point d coincide on the point b.

Translated bodies Velocity diagram

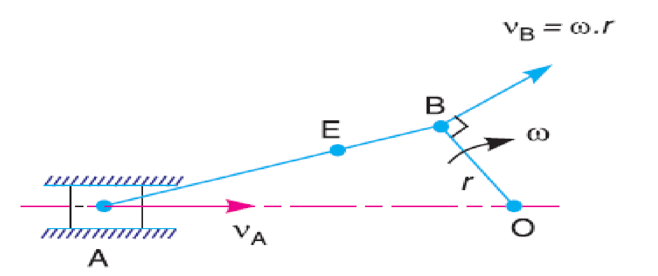
- ∵ the motion is absolute, then select any fixed point such as o be as a reference point (i.e point of zero velocity).
- Draw the path of translation.
- If v_B is known, select a scale factor to draw the velocity diagram (denoted by SFv)

$$SFv = \frac{draw\ value\ in\ mm}{actual\ value\ of\ velocity\ in\ (m/s)} = \frac{ob}{v_B}$$

The draw a line ob=(v_B)(SFv) in direction of v_B parallel to the path of translation.

Velocities in Slider Crank Mechanism

- The same method may also be applied for the velocities in a slider crank mechanism. A slider crank mechanism is shown in Fig. 5 (a).
- The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ὠ rad/s.
- Therefore, the velocity of B i.e. vB is known in magnitude and direction.

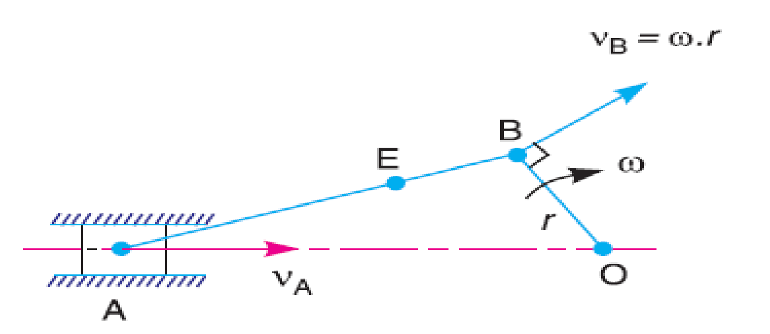


(*a*) Slider crank mechanism. **Fig. 5**

Velocities in Slider Crank Mechanism

- The slider reciprocates along the line of stroke AO. The velocity of the slider A (i.e. vA) may be determined by relative velocity method as below:

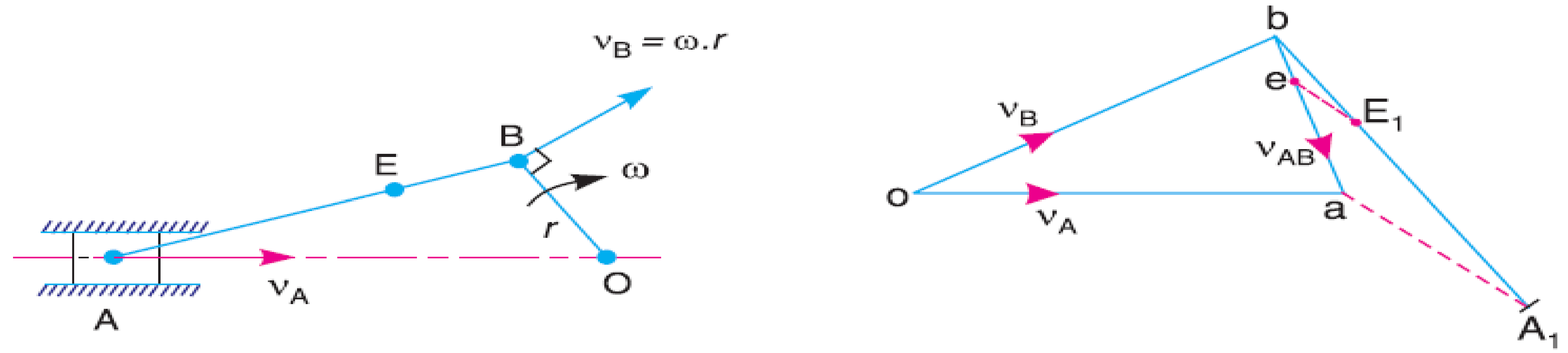
1. From any point o, draw vector ob parallel to the direction of vB (or perpendicular to OB) such that ob = vB= ὠ.r, to some suitable scale, as shown in Fig. 5 (b).



(a) Slider crank mechanism. **Fig. 5**

Velocities in Slider Crank Mechanism

2. Since *A B* is a rigid link, therefore the velocity of *A* relative to *B* is perpendicular to *A B*. Now draw vector *ba* perpendicular to *A B* to represent the velocity of *A* with respect to *B* *i.e.* v_{AB}.



(*a*) Slider crank mechanism. **Fig. 5** (*b*) Velocity diagram.

Velocities in Slider Crank Mechanism

3. From point *o*, draw vector *oa* parallel to the path of motion of the slider *A* (which is along *AO* only). The vectors *ba* and *oa* intersect at *a*. Now *oa* represents the velocity of the slider *A* *i.e.* v_A, to the scale. The angular velocity of the connecting rod *A B* ($\dot{\omega}_{AB}$) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

The direction of vector *ab* (or *ba*) determines the sense of $\dot{\omega}_{AB}$ which shows that it is anticlockwise.

Example

- *In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD =60°.*

Example

- **GIVEN :**

NBA=120 r.p.m

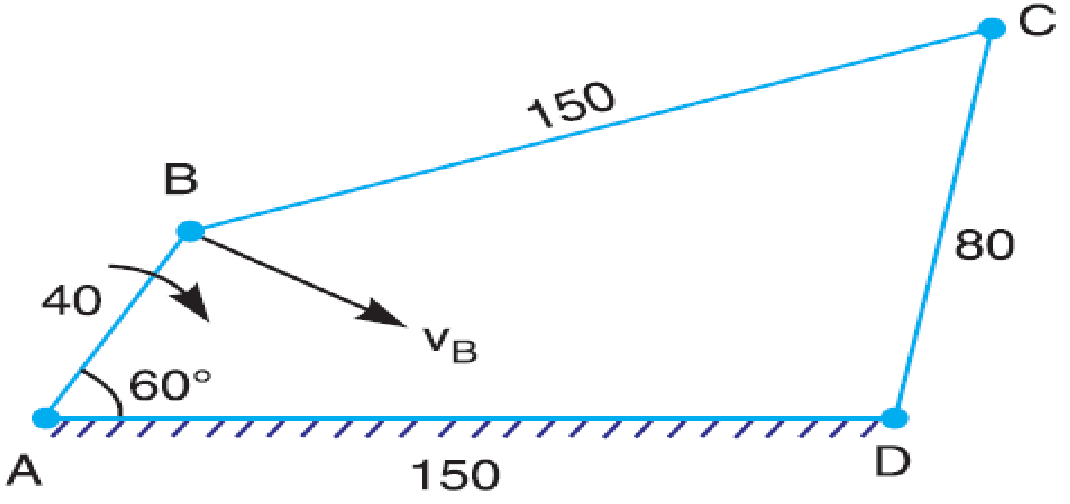
ὼ=2π×120/60

=12.568 rad/s

BAD =60°

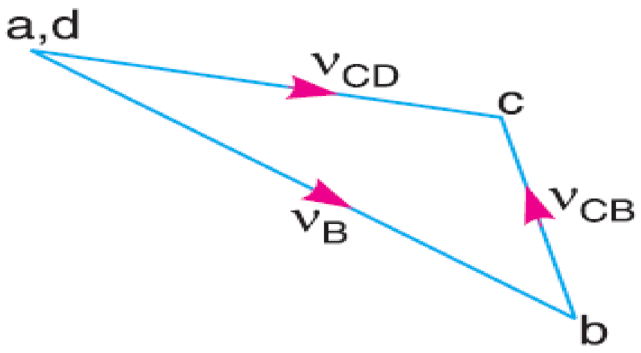
CD = 80 mm

Example



(a) Space diagram (All dimensions in mm).

Fig.6



(b) Velocity diagram.

Example

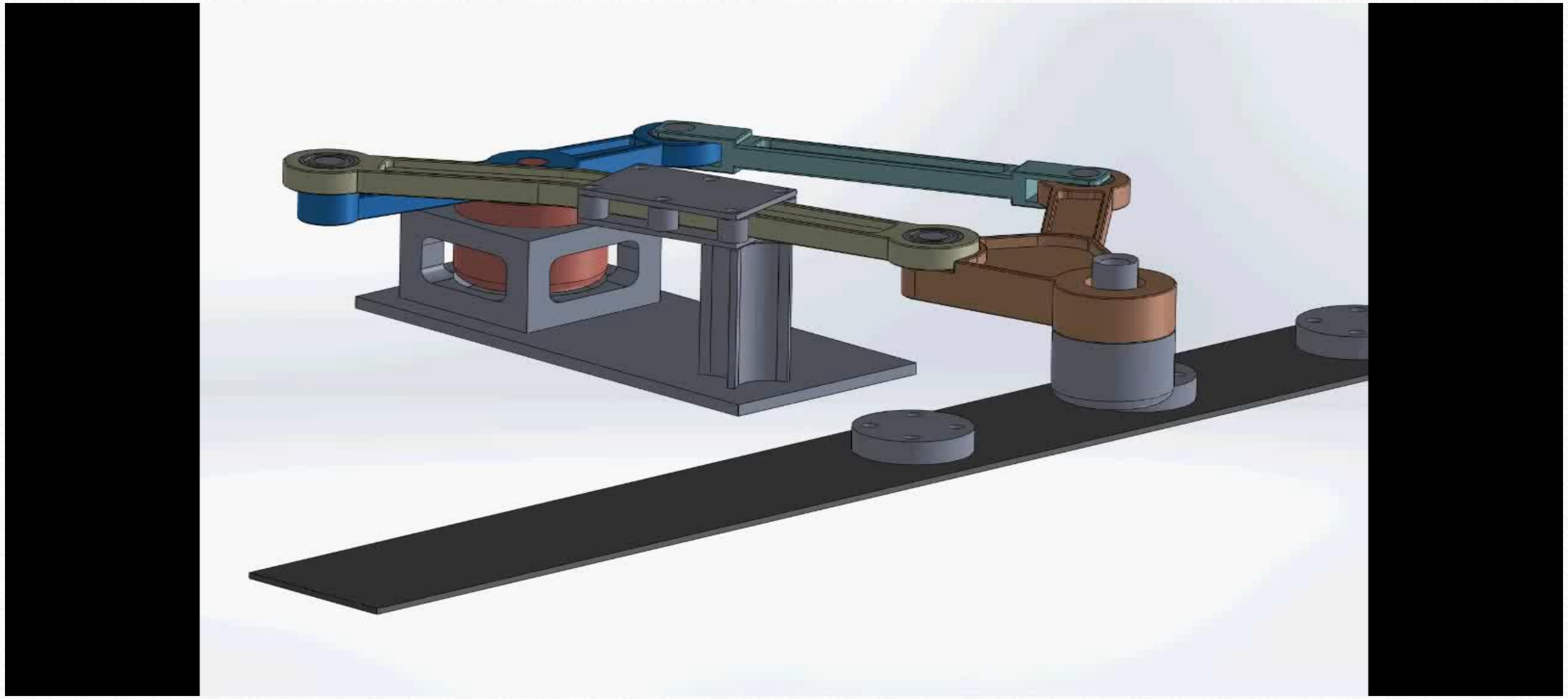
- ***SOLUTION:***

Since the length of crank *AB = 40 mm = 0.04 m, therefore velocity of B with respect to A or* velocity of *B, (because A is a fixed point), vBA= vB=ὼBA× AB = 12.568 × 0.04 = 0.503 m/s*

First of all, draw the space diagram to some suitable scale, as shown in Fig. 6 *(a). Now the velocity* diagram, as shown in Fig. 6(*b*)*, is drawn as discussed below*

Model Examples

- Application



Model Examples

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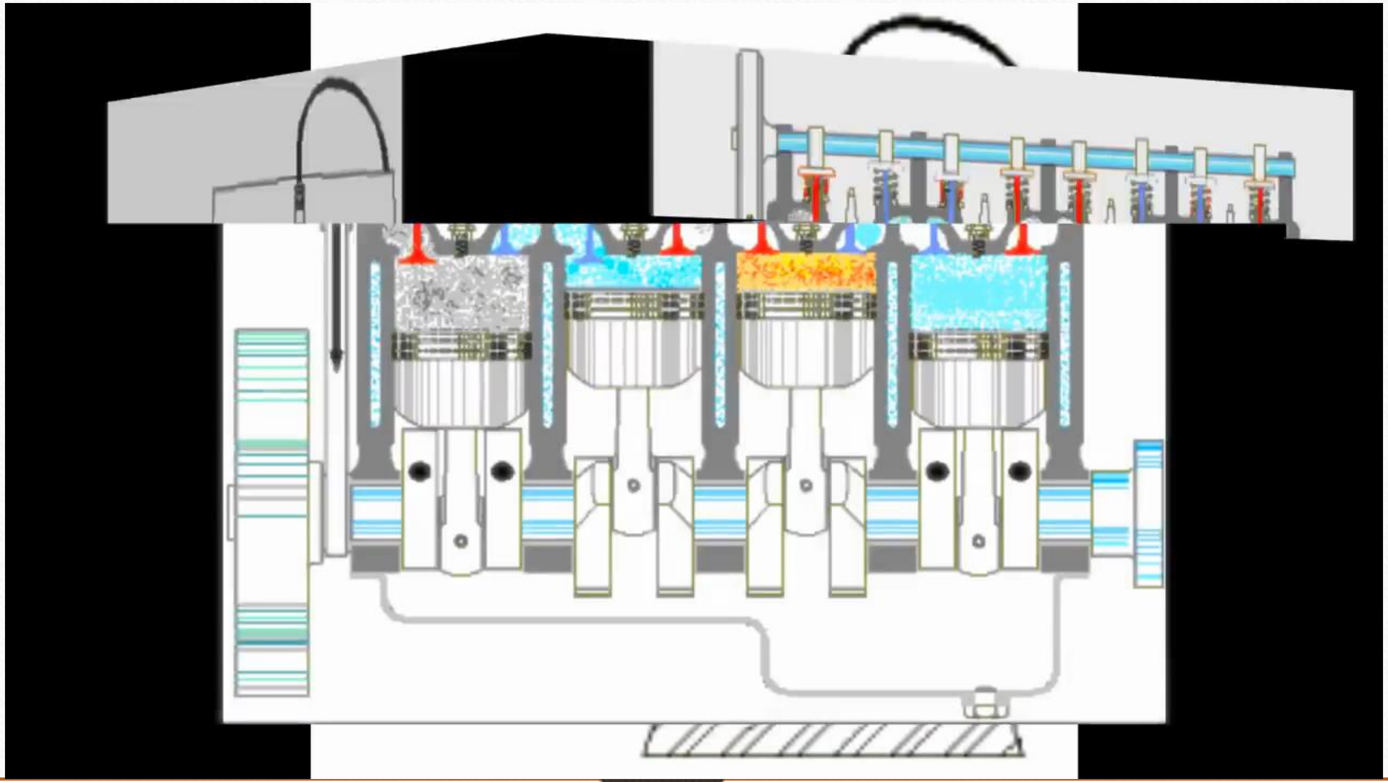

Thank you